A Continuous Profilometer

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During the course of investigation of mass transfer rates through boundary layers a means for measuring the depth of recession of a subliming surface was needed. Previous researchers have used three basic equipment designs for surface profile measurements.

The first method is that of placing a dial gauge at specific points on the solid surface before and after an experimental run. The difference in gauge readings is a measure of solid sublimation. Christian and Kezios (1959) employed this technique in their study of laminar mass transfer from sharp-edged cylinders. A micrometer dial indicator with 2.54×10^{-4} cm graduations was mounted on a lathe bed. Their procedure was to measure the radius of the cylinder at various axial positions before and after exposure to air in a wind tunnel. The cylinder was removed from the lathe position for the test exposure.

Thomas (1965) also used dial gauges in his boundary layer study. He mounted three micrometer gauges on a single steel bar and placed the bar transversely across a flat naphthalene surface at fixed distances from the leading edge to obtain depth readings at these points before and after each test.

Sherwood and Träss (1960) were the first to obtain a continuous profile of a subliming substance on a flat plate. They mounted strain gauges on a pick-up needle which was fastened to a movable bridge supported at the edges of the test plate.

The third design is that created by McLeod and Stewart (1960) and McLeod et al. (1962) who forced air through a hypodermic-type pin gauge, measuring air pressure to determine the distance between the pin end and the surface.

In review, each of the measuring techniques mentioned above has serious drawbacks. With dial gauges it is only possible to get point representation of the subliming surfaces. There is also a high probability of surface damage. Strain gauges give an excellent profile of the surface, but with light contact they are relatively insensitive and any increase in needle pressure in an attempt to increase their sensitivity might result in surface gouging. A disadvantage of McLeod's method, as mentioned by the authors, is the subjecting of the spot below the hypodermic pin to a jet of air which adds to the sublimation. The optimum instrument, therefore, should be able to follow continuously the surface profile of the subliming surface with high sensitivity and cause negligible damage to the surface.

The profilometer described here used a linear variable differential transformer (LVDT) as its sensing element. The LVDT (Figure 1) is an electro-mechanical transducer which transmits an AC voltage proportional to the linear displacement of its moving armature from the electrical center. An Atcotran No. 6234A-05B-0IXX with a linear displacement range of 0.254 cm at 0.05% linearity was used. The armature was mounted on a 10 cm long sharp pointed rod. When travelling over the surface, the LVDT rod, that is, the armature extension, slid vertically through two loose Teflon glands and was prevented from rotating. The total weight on the subliming surface was 12.322 grams.

The LVDT was mounted in the middle of a 25.5 cm $\times$ 3.75 cm $\times$ 2.54 cm Plexiglass arm (Figure 2). One end of the arm was pinned to a motorized carriage, and the other end contained a bearing which rode on a support plate for vertical stability. The tip of a ball point pen was used as a small bearing. The ink from the pen was flushed with solvent and replaced with a light lubricant oil. The LVDT support arm was attached to a drive screw activated by a high torque motor allowing a constant travel speed of 16.5 cm in 15 seconds.

Since it was desired to use a high speed recorder as the LVDT read-out device, it was necessary to rectify the output voltage from the LVDT. A full wave rectifier was constructed. The rectified wave was fed to a recorder whose span was adjusted so that for every 2.54×10^{-3} cm of armature movement, the recorder pen would move about 28 scale divisions. Changes in depth of at least 2.54×10^{-4} cm could be measured.

A typical output is shown on Figure 3. The upper trace

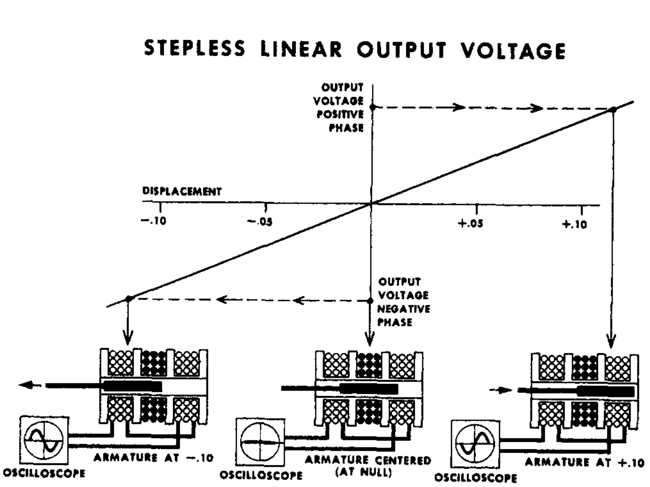

Fig. 1. LVDT schematic.

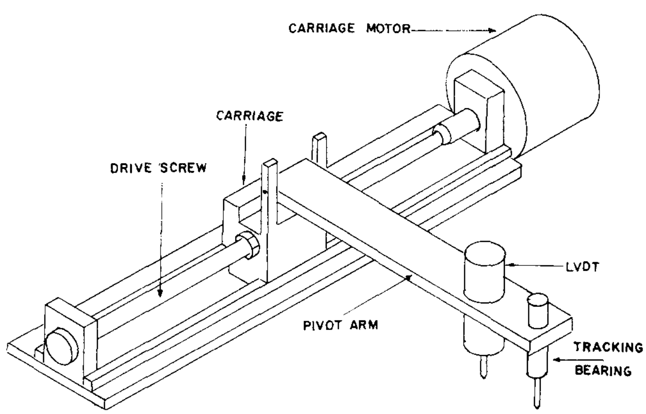

Fig. 2. Schematic diagram of profilometer.

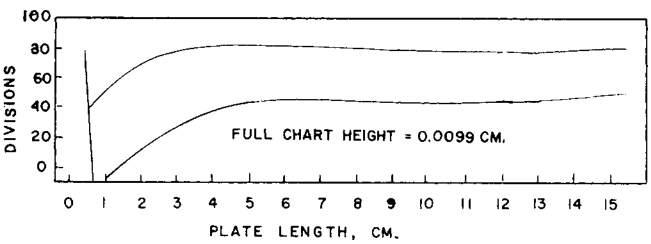

Fig. 3. Typical chart reading from profilometer.

is the surface of a naphthalene plate prior to its exposure to warm air. After sublimation for a period of time the surface was again measured and is shown as the lower trace. Thus, the difference between the two lines can be used as a direct measure of the amount of naphthalene that has sublimed.

The reproducibility achieved with this extremely sensitive profilometer was excellent. This profilometer lends itself nicely to continuous measurement of surface contours and should be quite useful for certain types of mass transfer studies as well as studies which require a record of changing surfaces.

LITERATURE CITED

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Examples of the Use of the Initial Value Method to Solve Nonlinear Boundary Value Problems

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Nonlinear ordinary differential equations of boundary value type occur rather frequently in various fields of chemical engineering. Different methods have been suggested to solve the governing equations. Runge-Kutta method is the most generally used numerical integration scheme. Due to the nature of boundary conditions, it is necessary to guess appropriate missing initial conditions and to match the computed result with the known final condition. This is always done by the trial-and-error method. This method takes considerable computer time to obtain accurate missing initial conditions. The purpose of this note is to illustrate the initial value method as a quite different approach to attack this kind of problem. Initiated by Topfer, Goldstein (1957) and extended by Klamkin (1962) and Na (1967, 1968), the method provides a very convenient alternative. By this method the original boundary value problem is transformed into an initial value problem which in turn is easily solved by any numerical integration scheme. Two problems previously appeared in literatures are solved to illustrate the application of this method.

POWER-LAW FLUIDS FLOW PAST A SEMI-INFINITE FLAT PLATE

Hsu (1969) recently examined the steady two-dimensional boundary layer flow of power law fluids past a flat plate, using the methods of series expansion and steepest descent to solve the governing equation. The present study solves this problem by the initial value method in an exact way.

Following Hsu, the resulting equation and the boundary conditions after similarity transform are given by

$$f''' + f(f'')^{2-n} = 0 \tag{1}$$

and

$$f(0) = 0, \quad f'(0) = 0, \quad \text{and} \quad f'(\infty) = 1 \tag{2}$$

where the prime denotes differentiation with respect to η. Now we define the following new variables

$$f = A^{\alpha_1} \bar{f} \tag{3}$$

$$\eta = A^{\alpha_2} \bar{\eta} \tag{4}$$

where A, α_1, and α_2 are constants to be determined. Substituting Equations (3) and (4) into Equation (1) we obtain

$$\bar{f}''' + \bar{f}(\bar{f}'')^{2-n} A^{\alpha_1(2-n)+\alpha_2(2n-1)} = 0 \tag{5}$$

For Equation (5) to be independent on A we let

$$\alpha_1(2-n) + \alpha_2(2n-1) = 0 \tag{6}$$

Thus Equation (5) becomes